Journal of Structural Geology, Vol. 3, No. 1, pp. 1 to 17, 1981
Printed in Great Britain

0191-8141/81/010001-17 $02.00/0

Folds and the strain ellipsoid: a general model

J. E. TREAGUS and S. H. TREAGUS

Department of Geology, The University, Manchester M13 9PL, England

(*Received* 21 *July* 1980; *accepted in revised form* 23 *October* 1980)

Abstract—A new general model is proposed to account for non-orthogonal relationships of fold axes and axial surfaces to the axes of the finite strain ellipsoid. Descriptions in the literature of such relationships, e.g. plunge variations and transecting cleavage, are reviewed and the development of the model is outlined. The model is described, in which plane strain ($k' = 1$) or slight flattening strain ($k' = 0.71$) or slight constrictional strain ($k' = 1.85$) are imposed on a suite of layers with oblique orientations to the bulk strain axes ($X > Y > Z$). Folds will initiate perpendicular to the direction of shortening in the layer and develop as non-material lines moving through the layer by angular migration. At geologically realistic finite strains the folds will be transected by XY(cleavage). In slight flattening strain, folds are likely to be cylindroidal with axes not measurably oblique to XY. In slight constrictional strain, en-echelon periclines are predicted, with marked transection by cleavage: in some examples the interference of two directions of folding will give rise to severe plunge variations and even downward-facing. Although strain markers frequently indicate finite strains in the flattening field, this may result from the superimposition of slightly constrictional tectonic strain on compactional flattening.

INTRODUCTION AND REVIEW

MODELS of folding generally have one feature in common: the assumption of horizontal layering and layer-parallel principal compression. The ensuing folds are symmetrical, with axial surfaces perpendicular to the median and enveloping surfaces and bisecting the interlimb angles, as defined by Fleuty (1964). Such fold models may be geometric (Ramsay, 1967, Hudleston 1973), finite-element simulation (Dieterich & Carter 1969, Shimamoto & Hara 1976) or dynamic (Biot 1961, Ramberg 1963). These studies imply, by analogy, that rocks were deposited and remained in regular horizontal layers and were deformed by a system of horizontal and vertical tectonic displacements.

The field geologist is faced with the problem of relating these models to real structures in folded rocks. Symmetric upright folds are rare; axial surfaces are commonly inclined to the median surfaces, as well as to the mean cleavage; non-plunging cylindrical folds are the exception. These relationships are clearly less simple than those predicted by theoretical models and require the development of other models.

A system considered to be of more general application to real structures was investigated theoretically and by modelling techniques (Treagus 1972, 1973): layers were considered to deform in compression, where the principal compression was not parallel to the layering. The intermediate strain, taken as zero, was parallel to the layering so that the layer was two-dimensional oblique. Two main results came from this work. The initial stresses should refract through layers of contrasting viscosity and thus infinitesimal strain should similarly refract and vary in size. Secondly, buckles should initiate symmetrically but become asymmetric by hinge migration and non-equal thickness changes on fold limbs, during finite strain. Anthony & Wickham (1978) have recently made a finite-element study of asymmetric folding, which supports the theory.

The two-dimensional oblique model may be applied to layers of sedimentary rock of inclined orientation, such as would exist in sedimentary basins deformed by major horizontal compression perpendicular to the strike of layers. The strain indicated by cleavage in this model would be expected to show variation around folds as a result of buckling strains, and refraction from limb to limb as a result of inhomogeneous layer strain. However, fold axes should remain parallel to the intermediate finite strain throughout the folding stack, and cleavages should refract about this axis so that they always intersect bedding parallel to the fold axis.

The two-dimensional oblique model is of general application to many folded areas where cleavage-bedding intersections are demonstrably parallel to fold axes and the mean cleavage is sub-parallel to axial surfaces. In many regions, however, detailed measurements show that this model cannot be applied in particular where fold axes are transected by the (assumed) contemporary cleavage. In these examples a more general three-dimensional oblique model is needed and is developed later in this paper. A brief review of recent literature concerned with such regions is given below, followed by a summary of the development of the general model.

Field descriptions

One of the earliest detailed structural analyses that revealed non-axial planar cleavage was that of Sutton & Watson (1956) in the Dalradian of NE Scotland. These authors showed that the strike of the earliest observed cleavage transected the strike of the axial planes of the first folds in two parts of a coastal transect; in one area the transection was clockwise, in the other anticlockwise, on either side of a third area where the cleavage was axial planar. They concluded that the cleavage was a superimposed structure, genetically unrelated to the folding. A recent examination of these exposures by one of us (J.E.T.) with Dr. J. Roberts has shown that a second cleavage is clearly superimposed upon the first folds, but that locally cleavage is oblique to folds in the manner described by Sutton & Watson; the cleavage fans and refracts, in sympathy with the folds, suggesting that they are genetically related. Hinges of first folds in this area commonly plunge up to 30° towards opposite quadrants, but some of the extreme plunge variations measured by Sutton & Watson (1956, plate III) are a mixture of first and later fold hinges.

Ramsay (1965) described a region of polyphase deformation in the Barberton Mountains of South Africa where the slaty cleavage crosscuts the first folds and is axial planar to the second folds. His observations support the view that one phase of deformation was followed continuously by a second, and cleavage developed from the summation of the two, thus crosscutting the first folds.

In Maine, U.S.A., Ekren & Frischknecht (1967) found that "where good exposures are available it is apparent that the cleavage is not parallel to the axial planes of the major folds. The cleavage therefore is younger than the major folds".

Brenchley & Treagus (1970) recorded first folds and cleavage-bedding intersections which exhibited strong plunge variation in a region of Ordovician rocks in Co. Wexford, Ireland. It was noted that the area presented special problems in the use of minor structures to deduce the major structure. Some of the variations of the cleavage-bedding intersection are now considered (J.E.T.) to arise from intersection of folds by non-axial plane cleavage.

Roberts (1971) described abnormal cleavage patterns in a region of folds in Northern Norway. He introduced a term 'arcuate hinge cleavage' to define the relationship recorded. The cleavage was synchronous with folding but its intersection with bedding curved across the fold hinge zones. A region of non-cylindrical and incongruous folds in Söröy, Northern Norway has been described by Ramsay & Sturt (1973). The folds have wavelengths of axial curvature at least 2–4 times the profile wavelengths, but frequently larger. One explanation proposed for this non-cylindrical folding was constrictional deformation. No associated cleavage or schistosity was described in the area. The terms incongruous and aberrant were used for minor folds with axes and axial planes which had an anomalous relationship to the major folds. Ramsay & Sturt suggested that these arose from asymmetrical relationships between fold and strain axes.

In an area of Precambrian rocks in Tasmania, Powell (1974) recorded variations of cleavage relationships to folds, both locally and regionally, from axial planar to cross-cutting. He introduced the term 'transected fold core' for the latter type. Powell sub-divided his folds into 'early' and 'late' of the same deformation episode and considered cleavage to develop over a relatively short time interval rather than continuously. The early folds predated cleavage formation and were cross-cut by cleavage, but the later folds were contemporary with the cleavage which was axial planar. The arguments for cleavage initiation over a short time interval are beyond the scope of the present review. Powell concluded that cleavage did not represent the finite strain plane XY, but that the axial planes were close to this plane.

Stringer (1975) described Acadian deformation in the N. Appalachians where cleavage was not parallel to the axial surfaces of folds. He observed variations across one fold where cleavage changed from axial planar to non-axial planar. His conclusions were that slaty cleavage was superimposed on the folds.

Borradaile (1978) described a new model for transected folds, with examples from Canada and Scotland. His theoretical model is discussed later. His Canadian examples are a reappraisal of Stringer's area (1975) following collaboration in the field, and depend largely on Stringer's data. Borradaile proposed a method of fold and cleavage development during the same deformation episode similar to Powell's (1974): cleavage developed relatively late in the strain history as a result of non-coaxial strain increments.

In the English Lake District, Soper & Moseley (1978) observed that cleavage was rarely exactly axial planar to the folds in the main Caledonian folding. The strike of cleavage was commonly 5–10° clockwise of the fold axial surfaces. They suggested that buckling began before cleavage formation and the discordance was a result of a rotational component of strain. Sanderson *et al.* (1980) have recently described a broad zone of non-axial planar cleavage in Lower Palaeozoic rocks, Central Ireland, adjacent to the Iapetus suture. Cleavage is clockwise in strike from the axial surfaces by up to 90°. No explanation was offered for the initiation of cleavage oblique to the axial surfaces, although the varying angle of transection is attributed to later simple shear.

The present authors observed folds cross-cut by a related fanning cleavage in the Silurian rocks of SW Scotland in 1969. It was tentatively suggested (Treagus 1972, p. 177) that this relationship might indicate folding of layers oblique to the three principal tectonic displacements. These preliminary observations were followed by a field study by J. Treagus and P. Stringer (Stringer & Treagus, 1980 and 1981). A coastal section of Ordovician and Silurian rocks about 50 km in extent was mapped, and 150 folds of intermediate size were recorded. Minor folds were rare. The structures were varied but the predominant factor observed was that most folds were markedly crosscut by cleavage. In longitudinal section the cleavage-bedding intersection commonly crossed the fold axis by 10–20°. However, in profile section cleavage fanned and refracted through particular layers and showed triangular fanning and occasionally finite neutral points around fold hinges (Ramsay 1967). These were considered irrefutable evidence of cleavage development synchronous to folding. Such relationships would not arise from late developments of cleavage in the same episode, or by superimposition of cleavage (cf. Rust 1965, Weir 1968). The folds in this area were found to be markedly irregular in size, shape and orientation: plunge variation was observed within particular folds and in adjacent folds across the whole region. The mechanism of formation proposed by

Stringer & Treagus supports the earlier model (Treagus 1972). Sedimentary layering was considered to buckle obliquely to the principal tectonic displacements so that the fold axes in the layers did not fall on the XY principal plane, later to become cleavage. This folding model has been placed in the framework of plate tectonic models for the region (Stringer & Treagus 1980, 1981).

The examples above demonstrate that regions of first folding with first (slaty) cleavage oblique to the fold axes or transecting the folds may be more common than generally accepted. Some of the interpretations above imply that these relationships are unusual or incongruous: this is because folds are expected to develop with axial plane cleavage. Indeed many explanations appear to be based on the argument that because the cleavage was not axial planar it must post-date the folding. Where the cleavage and folding were clearly both the earliest deformation in the rocks, and the cleavage was observed to show some relationships to folding, a mechanism was proposed where the cleavage reflected the later part of a rotational strain history. One theory (Powell 1974) proposed that cleavage development took place over a short time interval and therefore did not reflect the total strain in the rock. Yet another theory (Ramsay 1965) proposed a cleavage cumulative over two non-coaxial deformations. It is suggested here that the problem is simplified by viewing folding in rocks in a broader theoretical framework.

The development of the general model

Ramberg (1959) investigated the initiation of structures in a finite strain ellipsoid with principal axes $X > Y > Z$. He suggested that a thin competent layer would either shorten by buckling or stretch by boudinage according to its orientation. His examination included layers oblique to X, Y and Z and stated: "this axis of ptygmatic folds and the zones of... boudin fracture of such oblique veins... in general make oblique angles with the host rock schistosity and lineation". Flinn followed this with his classic paper (1962) on three dimensional finite strain. Flinn's method was to consider the movement of planes and lines in progressive homogeneous strain. He showed that the concept of finite homogeneous strain allowed for no folding or boudinage, but uniform behaviour of all layers. He thus assumed that folds generate by buckling, but are modified by homogeneous passive deformation.

Flinn presented a method of deriving the two principal strains in any plane in the strain ellipsoid by the Fresnel construction on a stereographic projection. From this, the directions of fold or boudinage generation could be determined (1962, p. 424): "the axis on which buckling takes place can be predicted from the deformation ellipsoid since it will be one of the two principal directions for that plane. What cannot be predicted is the attitude of the axial plane in a newly-generated fold. It is possible that the newly generated axial plane will be normal to the layer being folded. As soon as the axial plane is formed it rotates towards the largest axis of the deformation ellipsoid". Flinn also stated: "neither fold axes nor axial planes bear any special relation to the axes of the deformation ellipsoid". His conclusions in 1962 were that he would expect, in general, more complex relationships of folding and cleavage than were reported in the literature. He wondered if field data had been simplified in order to be more easily understood.

In earlier work with two dimensionally oblique layers (Treagus 1972, 1973) some preliminary suggestions were made about folding of generally oblique layers. It was stated that fold axes should not in general be parallel to a principal strain direction; that principal strains should refract from layer to layer in a contrasting multilayered sequence, such that intersections of principal planes would vary from layer to layer; and that buckles should initiate symmetrically but become asymmetric with development. Those ideas were subsequently developed, and recent work (Treagus, 1981) presents solutions for stress and strain variation in generally oblique layers in steady state flow. The results demonstrate that variations in size, shape and orientation of the infinitesimal strain ellipsoids, which occur according to layering attitude and viscosity contrast, should have no special relationship to the direction of infinitesimal folding. By geological analogy in finite strain, folds would thus develop with oblique and refracted cleavage of different orientation and bedding intersection in different layers. Current work (S.H.T.) is investigating the cumulative effect of steady state flow and buckling strains in a viscous layered system oblique to the three principal displacements.

The present paper, however, neglects the complicated feature of stress and strain refraction in layered rocks oblique to the principal compression. It examines the orientation of fold axes with respect to the bulk or mean principal strain axes: these are equivalent to the axes ($X > Y > Z$) of a homogeneous strain ellipsoid. In the generally-oblique layer the fold axis will be determined by the orientations of the two principal strains in the plane of layering as demonstrated by Flinn (1962). After folding this plane is represented by the envelope plane, and the fold axis should progress according to the positions of finite strain in the envelope plane. There are thus two distinct features of folding of this system: firstly the fold axis does not coincide with the intersection of the XY plane and layering; secondly the fold axis is not a material line and hence folding must progress by the interaction of passive fold-axis migration resulting from finite strain, and by active fold-axis migration indicated by the change in principal strain directions. These two features will be subsequently discussed. The present approach in terms of homogeneous strain does not conflict with the theories of stress and strain refraction since both theories require equality of strain ellipses on successive layer surfaces or envelope surfaces, in the multilayered sequence: this is a pre-requisite of continuity. The emphasis in this paper is on the range of variations of fold axes to the mean X, Y and Z directions, which might arise from buckling of different oblique layers, rather than the variation of X, Y and Z arising in a multilayered sequence.

Borradaile (1978) proposed a model of oblique layers in homogeneous strain to explain the occurrence of folds transected by cleavage in his two regional examples discussed above. Unfortunately some of his arguments and definitions are confused and the theoretical examples incorrect. He defined two angles for transected folds: "Δ is the dihedral angle between the cleavage plane and the fold axis" (the plane in which this angle is measured has not been stated) and "d is the angle between the axial plane and the cleavage in the plane of the fold profile". More precise definitions will be presented below. Borradaile (1978, figs. 3a and b) gave an example of an oblique plane and the derivation of its fold axis by Biot–Fresnel construction at two stages of deformation. This example is incorrect in two ways: firstly the two stages of deformation **a** and **b** are not on the same deformation path (Flinn 1965), having different k' values (our calculation from Borradaile's X/Y and Y/Z ratios). Secondly the orientation of the plane only increases in dip from **a** to **b** in Borradaile's example: its strike in the YZ plane does not change. This should only occur if deformation was purely constrictional ($k = k' = \infty$). Because of the error in position of the plane in his fig. 3b, Borradaile shows, incorrectly, the new fold axis with a smaller angle to the XY plane than the old fold axis. Both are sufficiently close in his example for him to consider them approximately the same: this should not be so. Moreover he claims "clearly fold axes initiated parallel to P_1 ... cannot remain parallel to the maximum extension direction in the enveloping surface in progressive deformation". This statement carries no justification.

Several methods may be used to derive the orientation of principal strains in a plane oblique to X, Y and Z. Flinn (1962) presented the Fresnel construction method on a stereographic projection, and Borradaile (1978) used a variation of this, the Biot–Fresnel construction. Ramsay (1967) used stereograms contoured for strain which gave immediate visual understanding and Ramberg (1976) presented a method of determination of the principal strains in any plane intersecting the strain ellipsoid, by vector algebra. Ramberg & Ghosh (1977) have computed rotations of planes and lines in three-dimensional progressive deformation in matrix notation. These numerical methods are not readily applied to a simple problem.

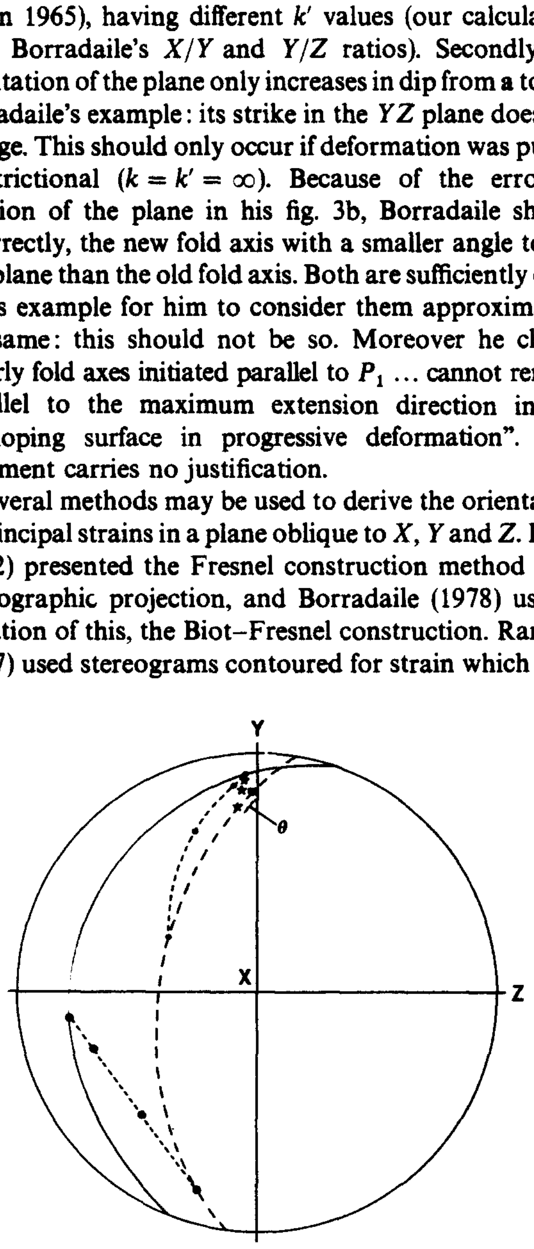

Fig. 1. Orientation convention of layering and strain axes $X > Y > Z$. The continuous great circle is initial layering; the broken great circle is deformed layering; large dots on the pecked line are the infinitesimal shortening directions in the layer for three strain increments; small dots on the pecked line are the infinitesimal extension directions in the layer for three strain increments; stars are the finite fold axes (maximum finite extension in layering) for three strain increments; the square is the deformed initial fold axis; θ is the axial migration angle. Lower-hemisphere Lambert equal-area projection.

THE MODEL

Plane layers oblique to the three axes of an irrotational deformation ellipsoid $X > Y > Z$ are assumed to deform in finite strain. The orientations of X, Y and Z are chosen to be X vertical, Y horizontal north–south and Z horizontal east–west (Fig. 1). Thus YZ is the horizontal plane and the dip of layering refers to its angle to this plane. The strike of layering is taken from Y. The results do not depend on this orientation convention: it was convenient for illustration, but also seemed the most probable general attitude of tectonic strains. Layers of 10, 20 and 30° dips and varied strikes are considered to represent the maximum range of sedimentary dips prior to folding. Higher dip values than 30° would probably indicate formation of an earlier structure to be refolded, or an exceptional sedimentary environment, so these have not been included in the results.

Three types of ellipsoid shape are considered, described by their k' value (Flinn 1965), where $k' = \ln(X/Y)/\ln(Y/Z)$. These are the plane-strain ellipsoid ($k' = 1$), a slight flattening ellipsoid ($k' = 0.71$) and a slight constrictional ellipsoid ($k' = 1.85$). Deformation is examined in three finite increments. For each ellipsoid type the same shortening value Z is used as a standard, and Y and X values as indicated by the k' value.

The oblique layers are considered to deform according to the strain ellipse in the plane of layering and to rotate towards the maximum extension X. The rotating and straining layers are assumed to generate folds in the competent members, with axes perpendicular to the direction of principal shortening in the layer. Layers are thus here considered to deform in two alternative manners according to their relative competence: by homogeneous strain or by buckling. This model lacks the refinement of strain variation and refraction in a multilayered sequence (Treagus, 1981) but simply considers a competent buckling layer in a homogeneously-deforming host. It may be considered to represent the relationship between the most actively buckled layers in a multilayered sequence and the mean or bulk strain ellipsoid, (X, Y, Z). The equivalence of the XY principal plane and mean cleavage is assumed throughout this paper.

The model of oblique-layer folding that we have described is of a layer whose fold axis at any stage is perpendicular to the direction of maximum shortening in the layer (or a plane representing the layer after folding, the envelope plane). In the oblique layer this direction is changing with strain and is not a fixed material line. The

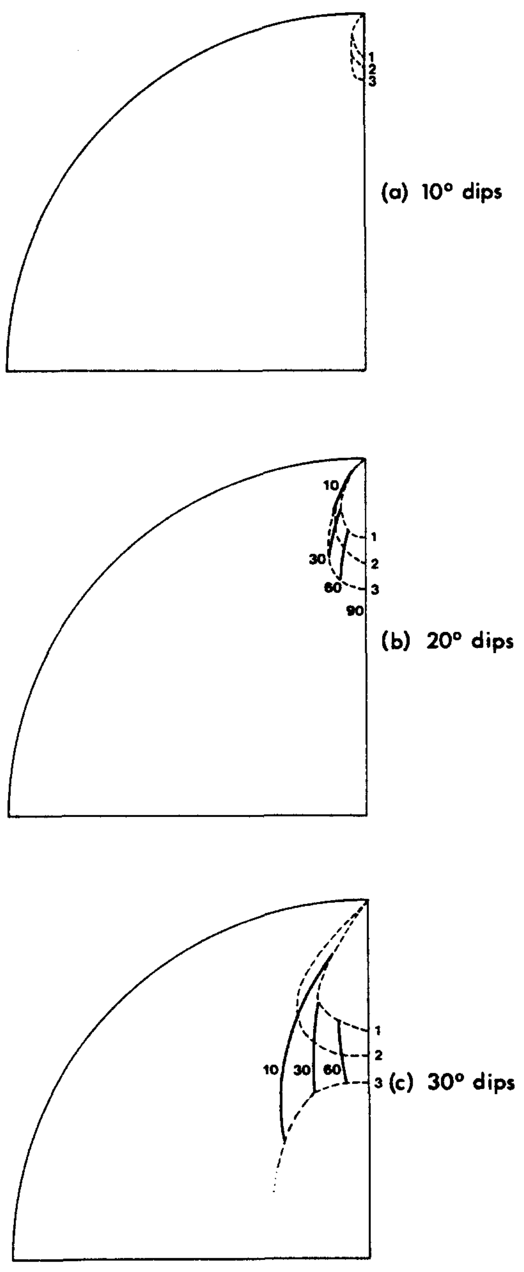

Fig. 2. Fold axis orientations in suites of layers, dipping into the NW quadrant, in three increments of plane strain ($k' = 1$); (1) $X = 1.26$, $Y = 1.0$, $Z = 0.79$; (2) $X = 1.59$, $Y = 1.0$, $Z = 0.63$; (3) $X = 2.0$, $Y = 1.0$, $Z = 0.5$. (a), (b) and (c) show initial dips of layering of 10, 20 and 30° respectively. In each figure the locus of all fold axes for each increment is shown by a broken line, and each increment numbered. Heavy lines on (b) and (c) show the locus of fold axes with strain, for layers of initial strikes (to Y) of 10, 30, 60 and 90°, numbered on the figures. Lower-hemisphere Lambert equal-area projection.

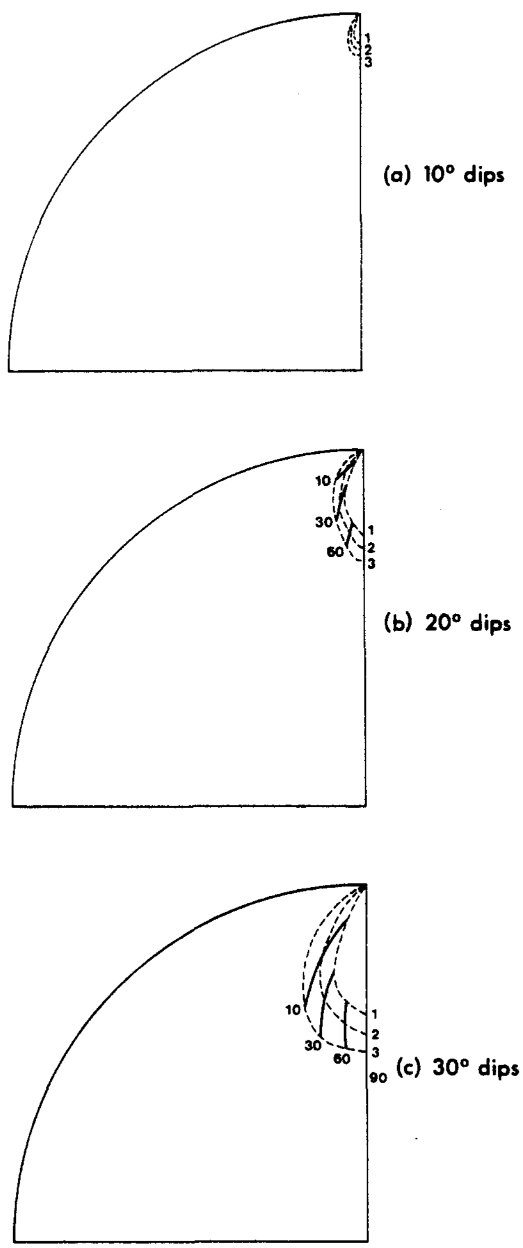

Fig. 3. Fold axis orientations in suites of layers, dipping into the NW quadrant, in three increments of slightly-flattening strain ($k' = 0.71$): (1) $X = 1.23$, $Y = 1.03$, $Z = 0.79$; (2) $X = 1.51$, $Y = 1.05$, $Z = 0.63$; (3) $X = 1.86$, $Y = 1.08$, $Z = 0.5$. Key as Fig. 2.

relationship between the fold axis direction, the infinitesimal fold axis and the deformed initial fold axis is shown in Fig. 1. The fold axis will not, in this model, behave as a passive line moving towards X as suggested in Flinn's (1962) model and later by Sanderson (1973) and Ramsay (1979).

We consider that folds are mobile structures for a considerable part of their development. They develop in oblique layers by oblique hinge migration in pace with the change in principal axes in the layer. The reason for the occurrence of buckling, that is a strong competence contrast, negates arguments for treating initiated buckles as part of a homogeneously deforming packet: this would imply a sudden change from active to passive behaviour which has no mechanical basis. We argue, too, that if buckles in oblique layers did not develop by oblique migration as indicated (Fig. 1) but by passive rotation of fold axes, fold axes would not be measurably oblique to XY. However this model of oblique fold accretion does pose certain problems which will be examined in the discussion.

In the Appendices, two methods are presented for determination of the direction and size of the principal strain ellipses in oblique layers. The equations of quadratic extensions in two and three dimensions (Ramsay 1967) have been combined. Appendix I gives a method for layers of known initial orientation: it was used in a computer program to obtain the results discussed below.

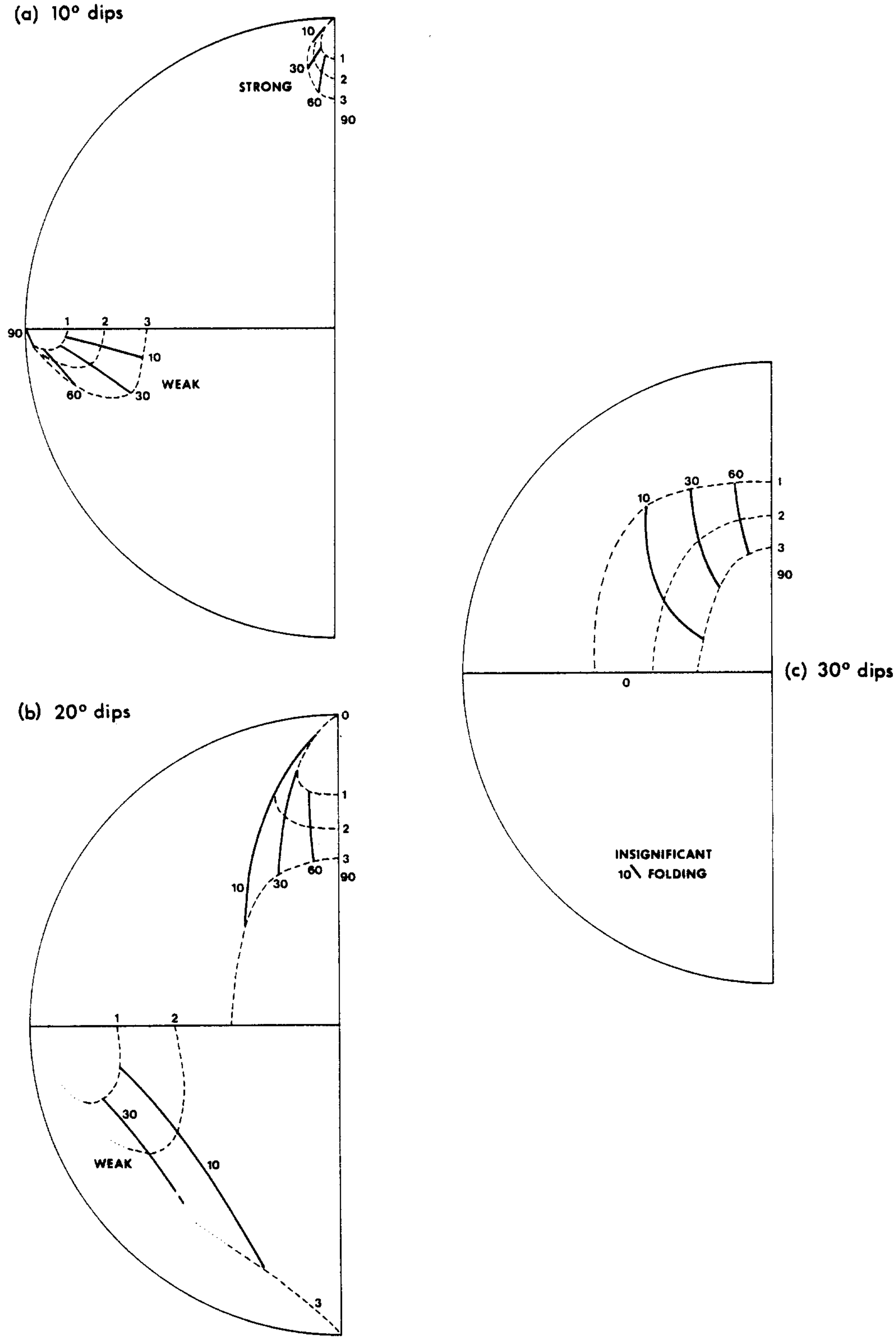

Fig. 4. Fold axis orientations in suites of layers, dipping into the NW quadrant, in three increments of slightly-constrictional strain ($k' = 1.85$): (1) $X = 1.33$, $Y = 0.95$, $Z = 0.79$; (2) $X = 1.76$, $Y = 0.9$, $Z = 0.63$; (3) $X = 2.33$, $Y = 0.86$, $Z = 0.5$. Key as Fig. 2.

Appendix II gives a method for layers of known deformed orientation; this is particularly useful where the sheet dip of a fold train (the enveloping surface) can be measured with respect to X, Y and Z.

RESULTS

Obliquity of fold axes to the XY *plane*

The positions of fold axes in the suites of planes of initially 10, 20 and 30° dips and varying strike, are presented for the three types of strain ellipsoid in Figs. 2–4. Three deformation increments are shown. The field of fold axes can be seen to relate to the initial dip, the amount of finite strain and the k' factor of the ellipsoid. Thus the obliquity of fold axes to Y and the XY plane increases from (a) to (b) to (c) in each figure and increases from flattening-type to constrictional-type strain. It can be seen that the locus of a fold axis with strain (heavy

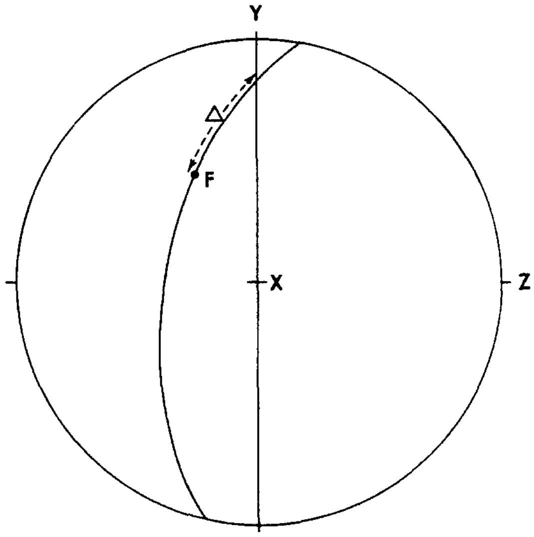

Fig. 5. Transection angle Δ defined for a generally-oblique layer (great circle) with a fold axis F (solid circle).

lines), for a particular initial strike and dip, does not approach the XY plane in the manner shown by a passive line (Fig. 1).

The degree of obliquity can be measured by the transection angle Δ following Borradaile (1978). This is the angle between the bulk XY plane and the fold axis, measured in the plane of deformed layering: the latter is equivalent to the enveloping surface, or median surface after folding. Δ is defined in Fig. 5 and values for Figs. 2–4 are graphed in Fig. 6. The increase in Δ from flattening to constrictional strain can be clearly observed. In plane strain and flattening strain where initial dips are 10–20°, the maximum Δ only exceeds 10° at $Z = 0.5$, or 50% bulk shortening.

The development of a fold in an oblique layer of particular orientation will depend on the changing magnitude of the principal strains as the layer rotates. Thus,

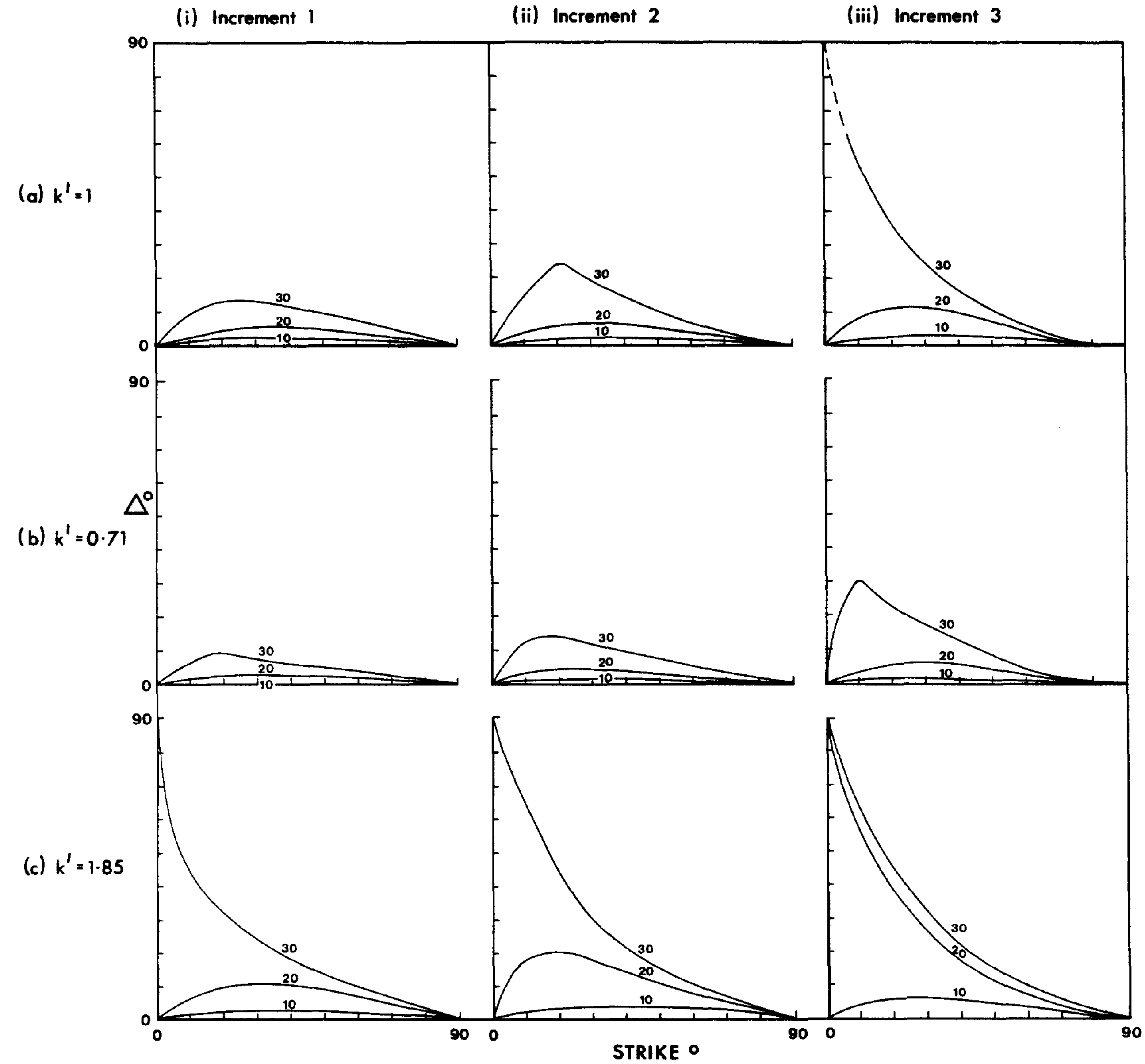

Fig. 6. Curves of transection angles Δ against initial strike of layering, drawn for suites of layers of initial dips 10, 20 and 30° (numbered on graphs), and for three strain increments. (a) $k' = 1$, from Fig. 2. (b) $k' = 0.71$, from Fig. 3. (c) $k' = 1.85$, from Fig. 4. Values of strain in each increment are given in captions to Figs. 2–4.

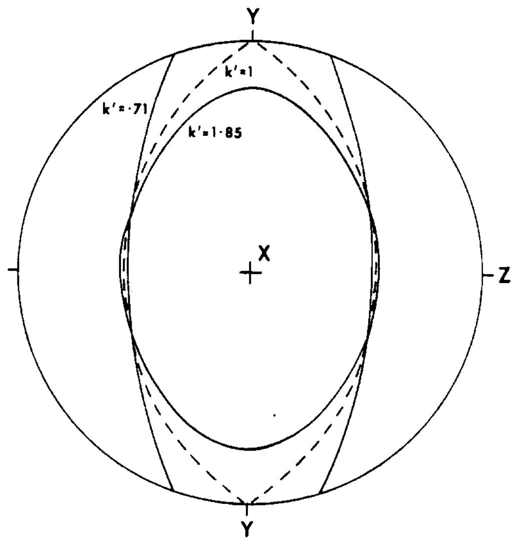

Fig. 7. Surfaces of no infinitesimal longitudinal strain for the three ellipsoid types with k' values as shown. Lower-hemisphere Lambert equal-area projection.

folds may become 'stronger', 'weaker', may cease to develop, or may become unfolded or boudinaged (Fig. 2c). In exceptional cases two directions of folding may operate simultaneously (Fig. 4c); the initially 'strong' direction may, at some stage in deformation, decline and be overtaken by the initially 'weak' direction. The effect in detail of the interaction between two directions of folding is discussed in the subsequent section. In Fig. 7, the surfaces of no infinitesimal longitudinal strain are drawn for the three ellipsoid types. They may be used in conjunction with Figs. 2–4; if the normal to the fold axis, in the plane, is a direction of infinitesimal shortening, then folding is still active.

The general conclusion to be drawn from this simple interpretation of the model is that folds developed on oblique planes under geologically realistic conditions will not differ in orientation greatly from those developed on planes which are orthogonal with respect to X, Y and Z. Thus, for planes with initial dips of 10° and under, and subjected to plane strain or slight flattening (generally assumed for most deformation), fold axes may be expected to develop very low plunges and an obliquity to cleavage ($=XY$) which would be scarcely detectable in the field. However, we consider that some of the commonly recorded 'aberrances' of fold axes, their plunge and obliqueness to cleavage, may be explained by development of the general model. Firstly, we will show that angular axial migration of folds should lead to periclinal development; secondly, we will show that pseudo-flattening deformation can arise from slightly-constrictional tectonic strain if volume loss is assumed; thirdly, if constrictional deformation is allowed as a 'normal' situation for deformation, then strong plunge variations in natural folds may be readily explained.

Axial migration

It has been proposed that in the general model of folding where layering is oblique to bulk X, Y and Z, folding must develop by oblique accretion. At any instant the increment of buckling will be parallel to the increment of maximum shortening in the layer, which changes continuously. The incremental fold axis is not coaxial to the finite fold axis, as shown in an example in Fig. 1. The fold axis moves through the layer (represented by the enveloping surface) during its development and cannot be considered as a material line. The angular difference between the finite fold axis and the deformed initial fold axis is here termed the axial migration angle. It is denoted in Fig. 1 by θ.

The progressive development of θ with finite strain is shown in Fig. 8 for two 20°-dipping planes (030/20 and 060/20). Results are presented for the plane-strain ellipsoid ($k' = 1$, Fig. 8a) and the slightly-constrictional ellipsoid ($k' = 1.85$, Fig. 8b). The incremental change of θ is given in Fig. 8(c). The highest angle of axial migration occurs in the slightly constrictional strain ellipsoid in the plane at lowest strike to Y. In plane strain the axial migration angles will probably not exceed 10° during progressive strain, and in flattening strain the angle will be negligible.

The actual amount of angular axial migration is expected to be slightly less than shown in Fig. 8 since the

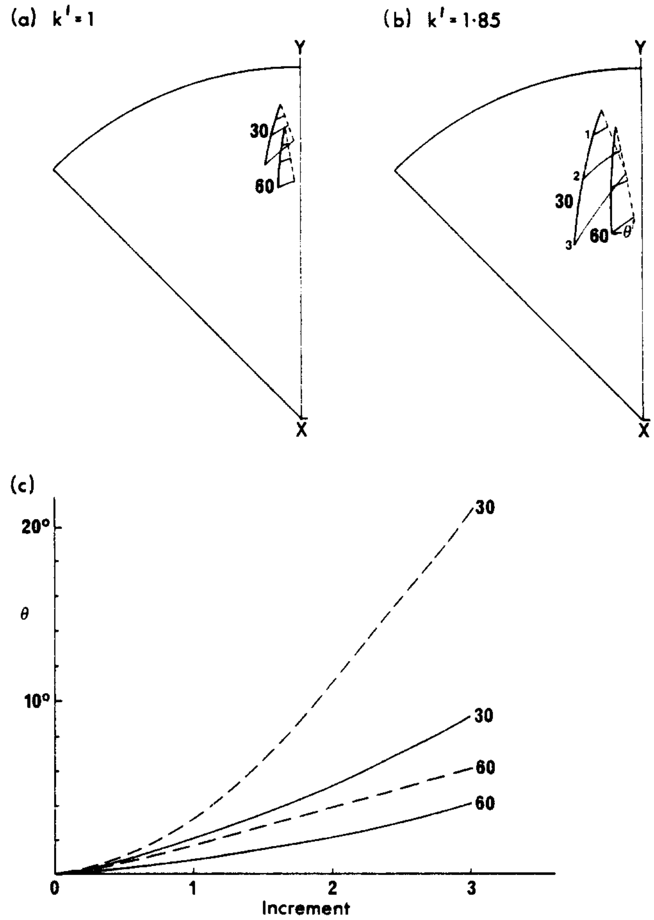

Fig. 8. Axial migration angle θ in layers with 20° initial dips and initial strikes 030 and 060°; numbered as 30 or 60 on each figure. (a) $k' = 1$, (b) $k' = 1.85$. Heavy curves are loci of fold axes; broken curves are loci of deformed initial fold axes; fine lines denote θ for the three strain increments (as numbered in (b) 30). (c) Values of θ from (a) and (b) for the three strain increments: solid curves $k' = 1$; broken curves $k' = 1.85$.

initial fold axis will not form instantaneously at the start of strain. In practice, therefore, θ will be the difference between the fold axis at the end of deformation, and the deformed position of the first-appearing fold axis. If the first significant fold axes have appeared after strain increment 1, the values of θ in Fig. 8(c) must be measured from this stage: they will be up to 2° less than the values shown.

This model of folding by oblique axial migration clearly differs from that of Flinn (1962) who treated fold axes as passive lines, and Borradaile (1978) who dismissed fold axis movements. It is also distinct from Sanderson's (1973) model to explain the obliquity of fold axes to Y in the XY plane. We consider that folding is the result of an active instability which progressively develops in pace with the strain in the layer. The angular migration of the fold axis during the development of folds will be as important as the changes in limb dip, shape and symmetry. Folding is thus a three-dimensional process in the most general case. Most previous models of folding have assumed that structures were cylindroidal: the development of folding could be treated in two dimensions (the profile plane), with the third dimension (the fold axis) parallel to Y of the bulk strain ellipsoid. Fold axes would thus be fixed and remain parallel. However, the recent buckling experiments of Dubey & Cobbold (1977), where layering was parallel to the YZ bulk principal plane (layer-parallel compression), have demonstrated some three-dimensional features of folding. Folding began as bulges at particular points of weakness, progressed to localised periclines with terminating axes, and finally developed to approximately cylindroidal folds on subparallel axes, during progressive strain. Fold axes thus developed laterally from a point source, via a restricted length to unrestricted length, by a combination of axial propagation and joining up of in-line folds. A change from folds dominated by initial irregularities to regular buckling was achieved. These layers had zero θ and therefore the fold axes could become fixed.

In generally-oblique layers the change from early periclinal folds to cylindroidal folds during progressive strain may be inhibited by axial migration. The processes of axial propagation and coalescence of in-line folds will not operate along fixed material lines as described for layer-parallel compression: their activity will be reduced in proportion to the axial migration angle. We therefore suggest that oblique layers which have a measurable axial migration angle ($\theta > 5°$) would rarely develop cylindroidal folds, and the change from folding dominated by initial irregularities to regular buckling would be incomplete. En-echelon periclinal folding centred at initiation sites with the periclinal axes moving in pace with the changing principal strain directions in the envelope plane, will develop (Fig. 9). The fold axes should be subparallel but plunge variations will arise from curving or branching fold axes, periclinal endings and truncation of abutting folds. A correlation between θ and the pericline length is expected: this is the subject of current work (S.H.T.). Pericline lengths 3–6 times the fold wavelength (axial wavelength/profile wavelength of 6–12) would be compatible with θ of 5–10°. Smaller ratios are indicated for $\theta > 10°$.

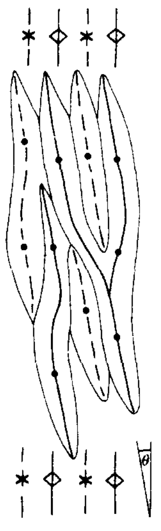

Fig. 9. Schematic development of periclinal folds of en-echelon and branching forms. Heavy lines are antiformal axes; broken lines are synformal axes. The orientation of the initial fold axes is given at the top and bottom. Initiation sites are shown by dots. Fine lines show the limit of final fold geometry. θ is the axial migration angle (= 10°).

In the examples of generally oblique layers in this paper (Figs. 2–4), θ values range from approximately zero to more than 20° according to layering attitude and strain ellipsoid type (Fig. 8). It is suggested that the value of θ dominates the geometric development of folds, to a spectrum of types from cylindroidal to periclinal. The value of θ broadly correlates with the transection angle Δ; thus a relationship between fold geometry and fold-axis obliquity to the XY plane exists. The oblique layers in the slight flattening ellipsoid ($k' = 0.71$) and some attitudes in plane strain ($k' = 1$) are expected to fold on axes subparallel to the XY plane but oblique to the stretching direction X, and develop approximately cylindroidal geometry. Some oblique layers in plane strain (initially moderate dip to the YZ plane and low strikes to Y), and the oblique layers in slight constriction ($k' = 1.85$) are expected to fold on axes oblique to the XY plane and develop en-echelon periclines. As the degree of fold axis obliquity increases, the periclinal wavelength approaches the profile wavelength. Considerable variation in fold axis azimuth and plunge is expected in these examples. The presence of two directions of folding in some examples will give rise to additional plunge variation as discussed below.

Strain with volume loss

The effect of volume loss during finite strain has been thoroughly investigated for a tectonic plane-strain model by Ramsay & Wood (1973). Volume loss could be divided into two types: early compaction of sediments (pure flattening in bedding) prior to tectonic strain; and tectonic strain with volume loss. The former process imposed a more dramatic effect on the resultant deformation. It was shown that total strain, or the 'apparent strain' of Ramsay & Wood (1973), indicated flattening. Thus strain data from deformed spots and other objects in slate belts indicating flattening ($k' < 1$) were shown to be

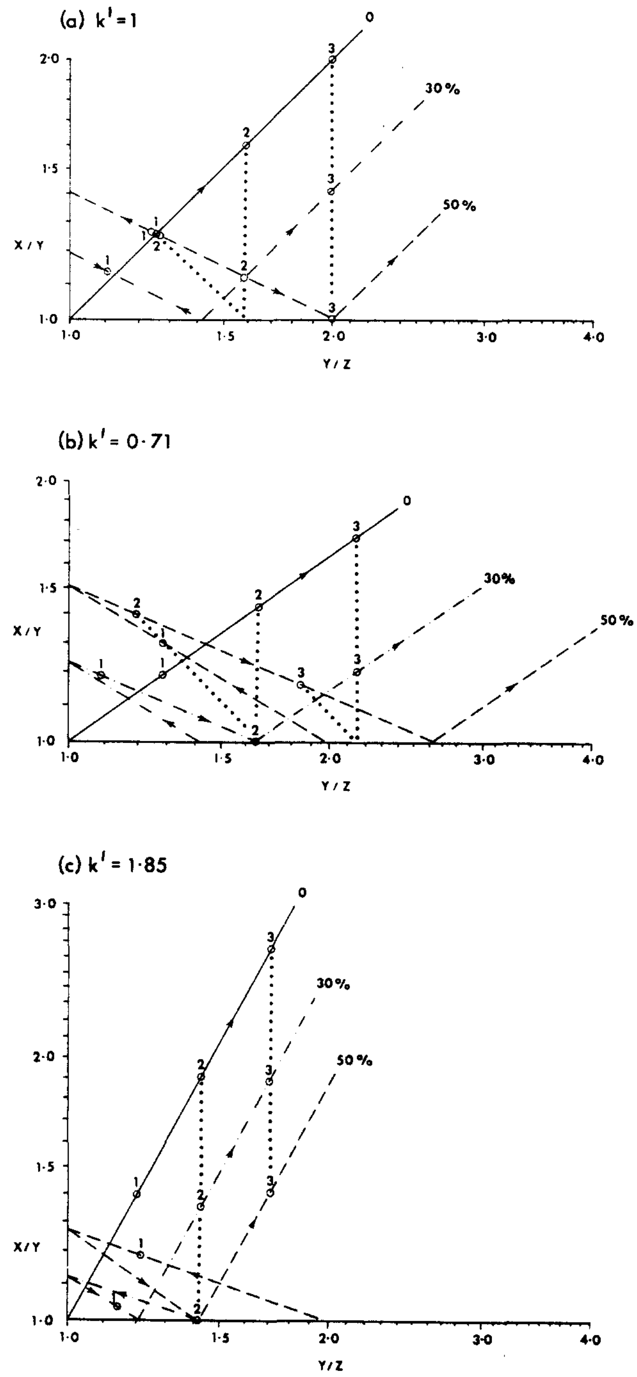

Fig. 10. Ramsay & Wood (1973)-type deformation plot X/Y against Y/Z (log. scale) for initial compaction with 0, 30% and 50% volume loss (solid, dot-dash and broken lines respectively), followed by tectonic strain of three ellipsoid types: (a) $k' = 1$; (b) $k' = 0.71$; (c) $k' = 1.85$. Arrows mark direction of deformation paths; open circles and numbers position each strain increment on the deformation path; dots mark the locus of total strain states with volume loss, after increments 2 and 3. Values of X, Y and Z for the three increments in each tectonic strain ellipsoid are given in the captions to Figs. 2–4.

consistent with initial compaction followed by tectonic plane strain.

The Ramsay & Wood (1973) method can be applied to tectonic strain ellipsoids which are not plane-strain, to determine the nature of the total strain for particular amounts of initial compaction. In Fig. 10 deformation plots of X/Y against Y/Z are shown for the three kinds of tectonic strain ellipsoid used in this paper, and for initial compaction of 0.30% and 50%. The positions of total strain for each of the three finite increments of tectonic strain are indicated. Figure 10(a) shows tectonic plane strain and the results are consistent with Ramsay & Wood (1973). In Fig. 10(b), the slightly flattening tectonic strain ellipsoid ($k' = 0.71$), the total strain field shows greater flattening than in Fig. 10(a), as expected. The slightly-constrictional ellipsoid is the most interesting graph (Fig. 10c). It is apparent that with initial compaction of more than 40%, total strain in the flattening field will result. This seems a reasonably conservative figure for compaction of sediments. It follows that strain markers indicating flattening could reasonably arise from slightly constrictional tectonic strain following earlier compaction strain. The strain markers and cleavage will result from the total strain, but the structural features such as folds and boudins will result from the tectonic strain only. Thus folded layers whose structures suggest a constrictional strain mode would be associated with cleavage and strain markers indicating flattening strain.

The examples in Fig. 10 were for three different tectonic strain ellipsoids in three equal increments of deformation. Comparison of the total strains for the three graphs shows that highest strains arise in the slight flattening case (Fig. 10b) and lowest in slight constriction (Fig. 10c). Thus the strain markers and cleavage in the latter case would be expected to display a lower X/Y ratio than the X/Y ratio indicated by two directions of folding.

No account has been taken here of volume loss during tectonic strain. The present treatment has not included the effect of dip of layering with respect to the tectonic strain axes, Fig. 10 assumed a simple co-axial relationship between the initial compactional strain and the tectonic strain. However it is probable that the total strain ellipsoid, represented by deformed markers and cleavage, and the tectonic strain ellipsoid, represented by folding and boudinage, are not only different in shape, but also in orientation, in a general folding model.

It has been shown by a simple model of compactional flattening followed by tectonic strain that the resultant total strain ellipsoid may markedly differ in shape and relative size from the tectonic strain ellipsoid. Ramsay & Wood (1973) were able to demonstrate that the strain data in slates which suggested flattening ($k' < 1$) could have arisen from tectonic plane strain. Plane strain is a convenient concept which may also be shown to be a good approximation to natural deformation. However it is also possible to argue that the strain data in slates could have arisen from tectonic flattening or slight constriction ($k' = 0.5$–2), from the results of Fig. 10. Thus apparent flattening indicated by cleavage and strain markers can theoretically be associated with constrictional tectonic structures in rocks with compactional volume loss. It seems reasonable, therefore, to suggest that folded rocks where the folds are markedly transected by cleavage (high Δ) and have axes strongly variable in orientation and plunge, have arisen from slightly constrictional tectonic strain. The mutual effects of two directions of folding will be examined below.

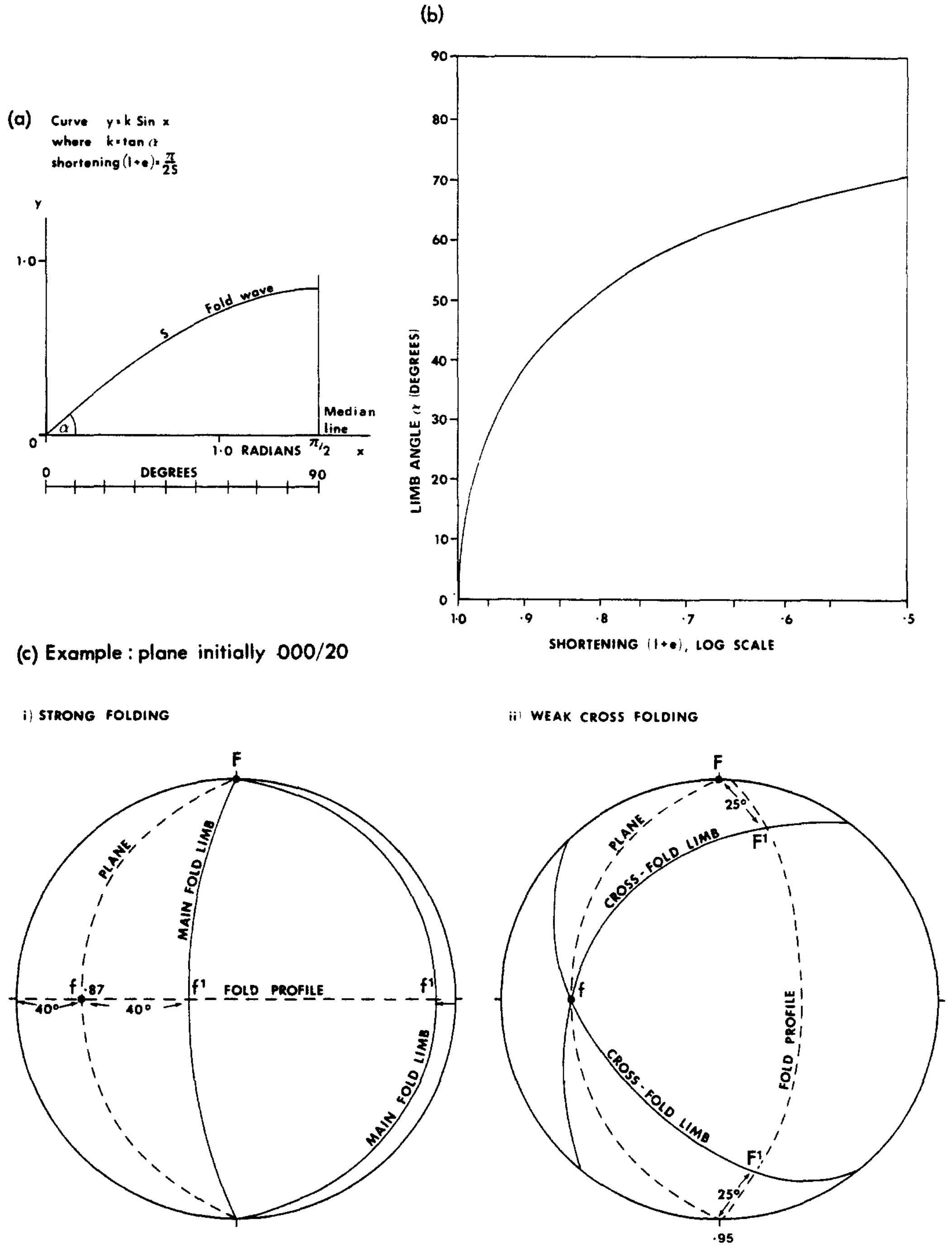

Fig. 11.(a)Geometric model for determining shortening in folding. α is the limb angle. (b) Relationship of shortening and limb angle α, derived from (a). (c) Use of the geometric model and graph (b) to determine the mutual effect of two directions of folding F and f (solid circles), in a layer (broken curve) initially 000/20°W, after one increment of slightly-constrictional strain (k' = 1.85: X = 1.33, Y = 0.95, Z = 0.79). (i) Strong folding on axis F with shortening strain $(1 + e) = 0.87$ and, limb angles 40°; f folded to f^1—f^1. (ii) Weaker crossfolding on axis f with shortening strain 0.95 and limb angle 25°; F folded to F^1–F^1.

Two directions of folding

The process of interaction of two simultaneous directions of folding will be examined by using two particular examples: planes with initial dip of 20° and strikes of 0 and 20° in the slightly constrictional ellipsoid (Fig. 4b). The amounts of the two principal strains in the layers for each increment will indicate the stronger and weaker fold axes.

In order to investigate the effect of one direction of folding on another, a given shortening strain must be converted into a fold of particular shape and limb dip. A model for folding must thus be introduced. A geometric model of fold growth with strain is considered to be more suited to the framework of this paper than the use of experimental or finite element results which are dependent on mechanical and mathematical premises. Folding is thus simply assumed to be sinusoidal, with a sine-wave form (Fig. 11a). All the layer shortening is taken up in folding, and thus a simple relationship between shortening and arc length exists. The shortening can be computed for waves of different limb angle (α) to the

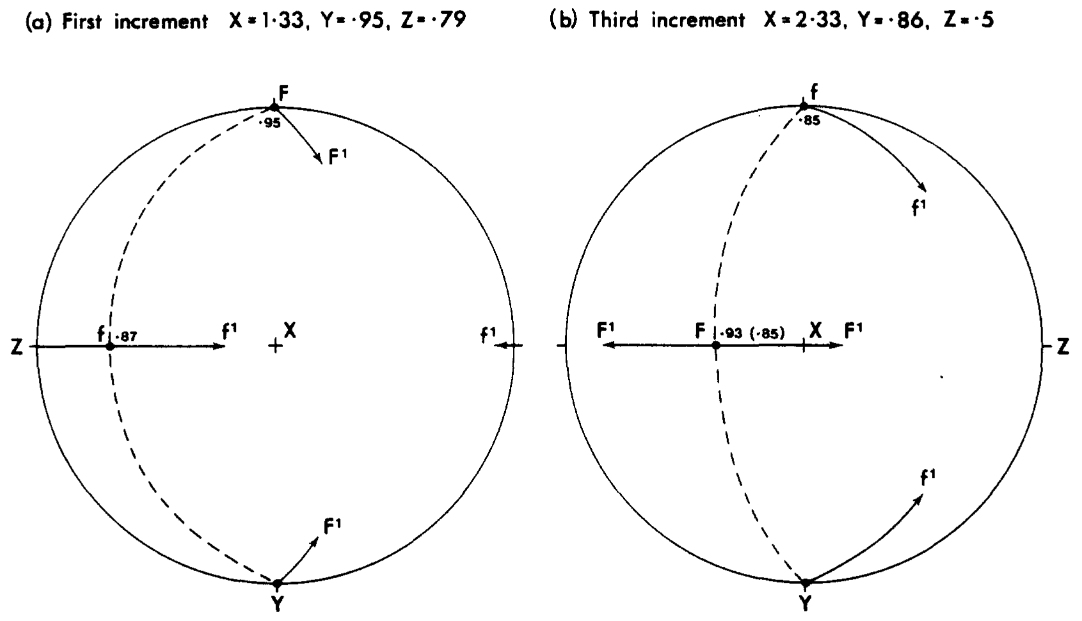

Fig. 12. Progressive effects of two directions of folding in layering (broken curve) initially 000/20° W, following Fig. 11. The two values of principal strain in the layer are marked; the maximum shortening undergone parallel to F is the figure in brackets in (b). F–F^1 is the main fold plunge variation, and f–f^1 is the crossfold plunge variation.

median line, (Fig. 11b). The graph enables an estimate of limb angle to be made for any shortening strain. This method has certain limitations. No layer shortening prior to folding is accounted for, so limb angles must be regarded as the maximum which might arise in the most actively buckling layers. Folding is assumed to be in symmetric waves, since the use of an asymmetric fold wave form would be dependent on assumptions about axial-plane orientation. In previous work (Treagus 1973) folding of oblique layers was considered to initiate symmetrically. Since the results below are confined to folds of predicted obtuse interlimb angles, fold symmetry seems a reasonable approximation.

An example of the use of Fig. 11(b) is given in Fig. 11(c) for a plane of initial dip 20° and strike zero (two-dimensional oblique). Two directions of folding are shown in Fig. 11(c), (i) and (ii): F with limb angles of 40° and f with limb angles of 25°, arising from the shortening values shown. Each fold might have an effect on the other as shown. The variation of the main fold axis F–F^1, arising from the weak cross folding of f, is more significant here than the variation f–f^1. In some examples the two directions may interchange in significance, or be equally strong, so the geological significance of the weak fold axes cannot generally be discounted.

The use of the method to investigate the interaction of two directions of folding during progressive strain is illustrated by two examples. Figure 12 shows the first and third increments of deformation of the initial plane 000/20 of Fig. 11(b). The two directions of folding are fixed, but interchange in significance as the shortening values show. The early main fold axis F becomes a direction of extension and the weak fold f remains a shortening direction throughout strain, so that it has become the stronger direction in Fig. 12(b). Where a change from shortening to extension has occurred, the maximum shortening undergone, shown in brackets in Fig. 12(b), has been used to compute plunge variation. The omission of fold modification in extension is justified below. Figure 12 is a good example of the change in importance of the two folding directions. The finite effects after the third increment are two directions of folding of equal limb angle with mutually-affected plunge variation. The folds would be expected to have a dome- and basin-type geometry. Downward facing occurs at F^1 as a result of plunge variation.

Figure 13 is an example of two directions of folding in a generally-oblique plane (020/20). The three-dimensional obliquity requires angular fold axis migration as described previously, in contrast to Fig. 12. The two migrating potential fold axes do not change in relative importance during finite strain. The strong folding direction F and the weak direction f should exist concurrently until the cessation of shortening in f. The fold axis f would not be expected to be apparent as such: its real importance is in its effect on the main fold plunge F–F^1. A high axial migration angle θ is indicated by the change in F from Fig. 13(a) to (b). Periclinal main folds would be expected to develop and be modified by the crossfolding f.

These examples, based on a simple geometric model of folding, are given to illustrate some potential features of interaction of folding, in slightly constrictional strain. The incorporation of extensional features is beyond the scope of the simple model. Extension in the form of decrease in fold limb dips is considered unacceptable: folding is here envisaged as irreversible. Fold modification by homogeneous strain is considered inappropriate too: it cannot be argued that a competent layer which actively buckles in compression will become passive in extension.

The effects of two directions of shortening in rocks are considered to give rise to significant plunge variations as illustrated theoretically. Main fold axes will be strongly

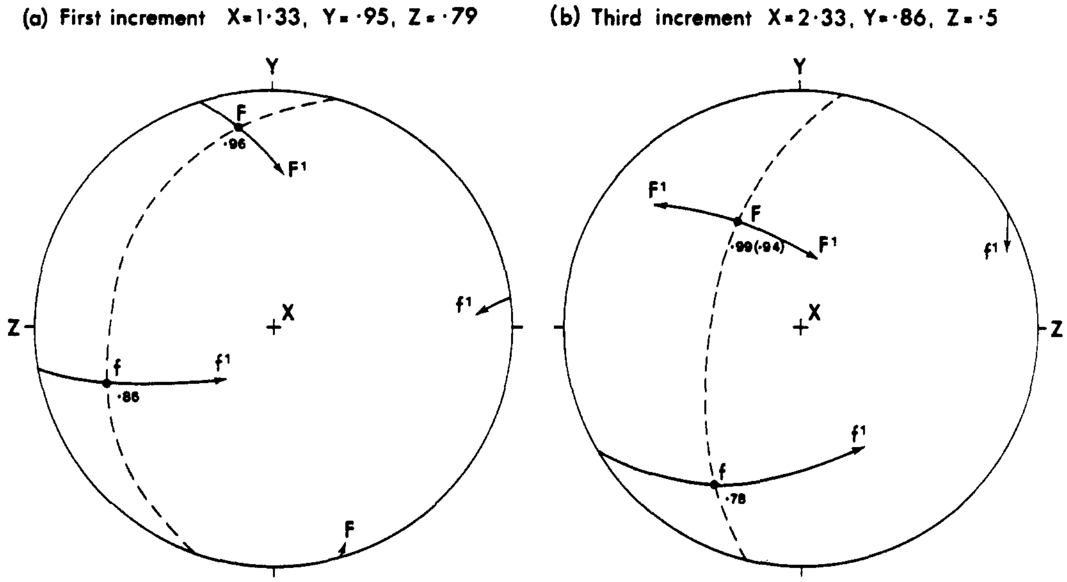

Fig. 13. Progressive effects of two directions of folding in layering (broken curve) initially 020/20° NW, following Fig. 11. The two values of principal strain in the layer are marked; the maximum shortening undergone parallel to F is the figure in brackets in (b). F–F^1 is the main fold plunge variation and f–f^1 is the crossfold plunge variation.

variable and generally markedly oblique to cleavage. Two other factors will contribute to the variation of fold axes, however. Since the layers have two principal directions of shortening, all lines on the bedding plane are directions of shortening and potential folding. Thus, if initial irregularities affect the positioning of folds, fold axes may arise in any direction between F and f. The competition between the strongest folding in direction F, and the use of inhomogeneities for fold siting, will control the regularity of folding. In layers with good bedding planarity, periclinal folding on axes subparallel to F, with plunge culminations and depressions arising both from periclinal noses and from crossfolding f, would be expected. Layers with irregular thickness, marked bottom structures and other inhomogeneities would be expected to display locally a wide variety of fold-axis orientations, in addition to the plunge variations arising from the factors discussed above. Here folding may locally be aberrant, downward facing and strongly transected by cleavage. The mutual interference of directions of folding in all orientations is shown schematically in Fig. 14 for three initial orientations of layering: layer-parallel compression (a); two dimensionally oblique (b) and three-dimensionally oblique (c). Areas of strongest folding are shown, and the region where fold axes have passed through the 90° pitch on their axial planes (perpendicular to layering in these examples) is designated 'downward facing'.

Natural layers in slightly constrictional strain are thus expected to show the following features: folds of periclinal form with axes oblique to cleavage; statistically variable fold axis orientation, locally and regionally, with a correlation between scatter and initial inhomogeneities of bedding planarity; apparent crossfolding with resulting culminations and depressions of plunge which may give rise to downward facing. The relative dominance of the theoretical periclinal wavelength indicated by the θ value, and the crossfolding wavelength, (equivalent to the main fold wavelength), cannot be stated conclusively. It is suggested that the periclinal process, which is proposed in this paper to be a significant factor of generally oblique folding, would dominate the early stage of folding and over-ride the initiation process of weak crossfolding. In practice therefore the plunge variation arising from crossfolding would most probably be manifested in strongly periclinal folding on variable axes.

The above features are consistent with many regional descriptions of unusual or aberrant relationships of folds and cleavage, some of which were outlined in the Introduction. We tentatively propose this model of folding in slightly constrictional tectonic strain as a working hypothesis for regions of strong plunge variations and local downward facing. It has been suggested as an explanation of the structures in the Galloway region of southwest Scotland (Stringer & Treagus 1980). It was the explanation most favoured by Ramsay & Sturt (1973) for the fold patterns in Söröy, north Norway.

CONCLUSIONS

An examination of strain in layers of general non-orthogonal orientation to the axes of the bulk strain ellipsoid has enabled a general model of folding to be proposed. Layering is restricted in initial orientation to low sedimentary dips, and tectonic strains are taken as X vertical and Y and Z horizontal. The bulk strain ellipsoid is not restricted to the plane strain case. This general folding model is shown to be associated with features which have previously been regarded as unusual and aberrant. These are summarised below.

(1) Fold axes will develop oblique to the bulk XY or cleavage plane. Cleavage is therefore non axial-planar to

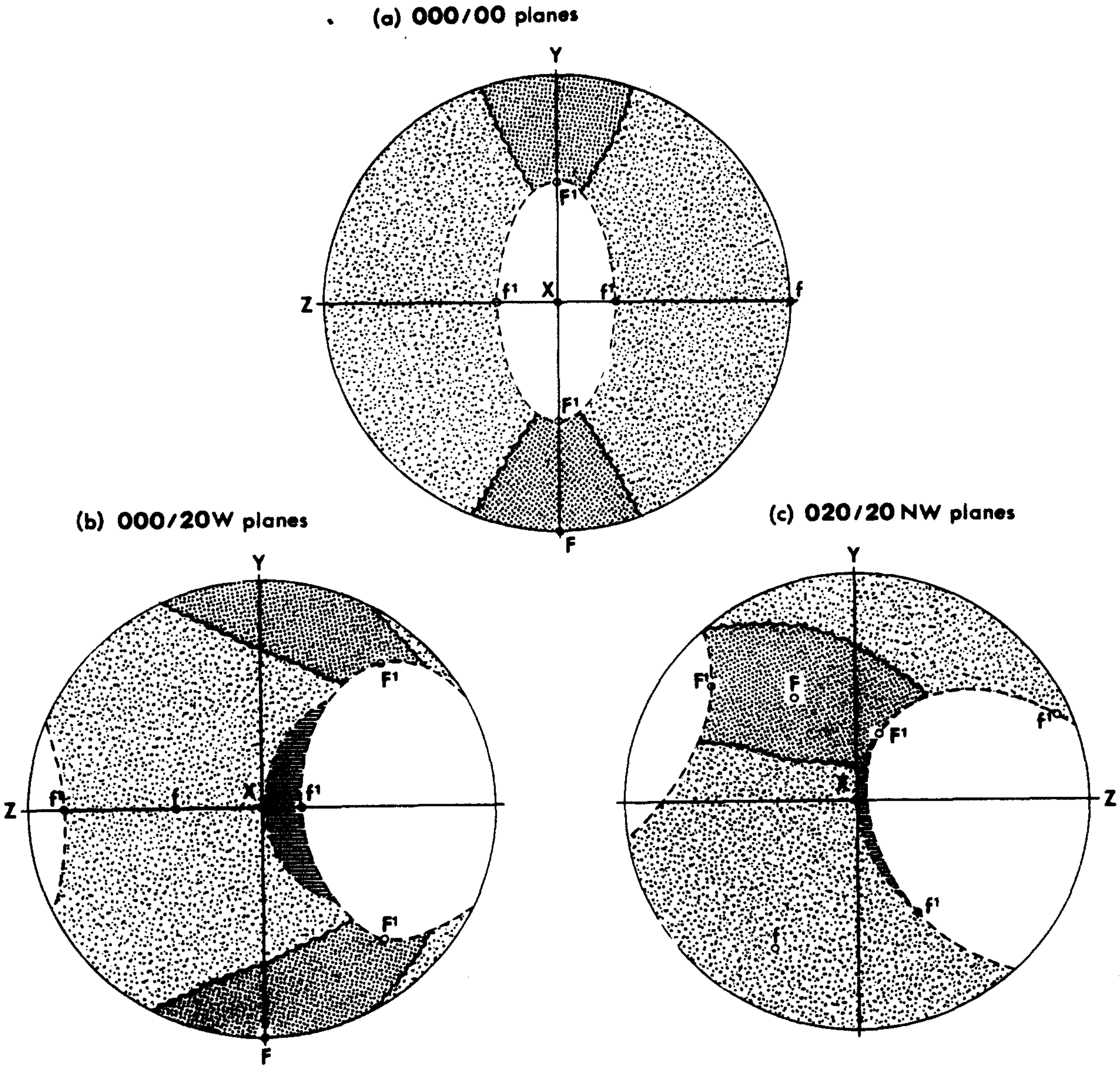

Fig. 14. Patterns of possible fold axis orientations after three increments of slightly constrictional strain ($k' = 1.85$; $X = 2.33$, $Y = 0.86$, $Z = 0.5$) for layers in three initial orientations: (a) 000/00 (horizontal); (b) 000/20° W; (c) 020/20° NW. Solid circles denote F and f, the principal fold axes. Heavy dotted shading is the approximate field predicted for the most active folding with plunge variation. Fine shading is the field of possible folding, with plunge variation. Lined shading marks regions of "downward-facing folds" defined in the text.

folds. The obliquity of fold axes to cleavage will be related to the k' factor of the bulk strain ellipsoid. In plane strain and slight flattening the transection angle will be small, and fold axes will be approximately parallel to cleavage, with a plunge. The cleavage-bedding intersections will therefore be approximately parallel to the fold axis. In slightly constrictional strain, however, transection angles will be more marked. If the layer is in total shortening, greater variation of fold axes will arise (see (5) below).

(2) Folding will progress by angular migration of axes such that the fold axis is a non-material line. The size of the axial migration angle (θ) will vary according to the strain state, initial layer attitude and degree of initial irregularities in bedding. In layers with zero θ (parallel to X, Y or Z), cylindroidal folds of indefinite length should develop. In generally oblique layers in flattening strain, and some orientations in plane strain (low dips to YZ, high strikes to Y), folding should be approximately cylindroidal on parallel axes. In generally oblique layers in slight constriction, and some orientations in plane strain (moderate dips to YZ, low strike to Y), folding should generally be non-cylindroidal. Periclines centred on initial irregularities, in en-echelon arrangement with branching or terminating axes, are expected. Subparallelism of periclinal axes is predicted for plane strain states, and curving variable axes for more constrictional strain states.

(3) The features of (1) and (2) above can be combined as a correlation of associated structural features and the tectonic strain state in which they arose (Table 1). The features form a spectrum from flattening to constrictional strain, but their relationship to layering attitude gives an extra dimension of variability not shown in Table 1. The degree to which the structural features in oblique layers in plane strain reflect more flattening or more constrictional features is dependent on initial attitude of layering X, Y and Z.

Table 1. Correlation of structural features with tectonic strain state

Flattening strain	Plane strain	Constrictional strain
Fold axes parallel to XY	Δ increasing	Fold axes strongly oblique to XY
Cylindroidal folds		Periclinal folds
Infinite fold length	Definable fold length	Periclinal wavelength
Axial-plane cleavage		Non-axial plane cleavage; transected folds
Cleavage-bedding intersection parallel to fold axis		Cleavage-bedding intersections vary around fold; oblique to fold axis.

The geological features summarised in Table 1 for flattening strain and some plane strain examples are those often reported in field descriptions. The only indication that those structures arose in layering initially oblique to XY and Z, rather than in layer-parallel compression would be the non-orthogonal relationship of the stretching lineation and fold axis or cleavage-bedding intersection, on the cleavage plane.

(4) Constrictional strain where Y is a direction of slight shortening has been shown to be reasonable as a tectonic strain model. The presence of cleavage and flattened objects in most folded rocks would seem to negate constrictional deformation. However tectonic constriction following compactional flattening of more than 40% would give rise to constrictional fold structures associated with a flattened fabric.

(5) The interaction of two directions of folding is suggested to give rise to strong plunge variations and even downward facing. These will be combined with the general feature of periclinal folding. Moreover since all directions in the layer are shortening, they are potential fold axes. Thus a wide field of main fold axes, affected by crossfolding to give plunge variation, would be expected particularly in layers with strong initial inhomogeneities. The essential geological features peculiar to strain with slight shortening in Y, are summarised below:

(i) Folding on strongly-varied axes; variation within single folds and in adjacent folds, laterally in the envelope plane and in plunge.
(ii) Folds with anomalous orientation with respect to others, or downward facing.
(iii) Periclines of dome-basin form.
(iv) Cleavage non-axial planar. Folds cross-cut or completely transected by cleavage. Cleavage-bedding intersections are noticeably non-parallel to fold axes in most folds, and thus appear to curve around the folded layer.

(6) All the features of folding and strain described previously refer to the bulk strain and bulk or regional cleavage. In detail the folds are expected to be associated with cleavage which varies in strength and orientation with rock competence, and varies in orientation in sympathy with folding. It is tentatively suggested that competent layers may contain cleavage in convergent fans (in profile) intersecting bedding approximately parallel to the fold axis, while the incompetent layers with divergent fans have cleavage strongly oblique to the fold axis. In certain fold regimes the effect of cleavage fanning as a result of folding, combined with the cleavage arising from oblique strain, may give rise to some unusual cleavage-bedding relationships, (Roberts 1971, Stringer & Treagus 1980).

(7) Many of the features outlined in these conclusions might be attributed to polyphase deformation. Some such explanations for regions of folds transected by cleavage were given in the Introduction. It is therefore imperative that we present criteria which may be used to distinguish structures of unusual types arising in a single phase of deformation from genuine superimposition of folding and cleavage formation.

If the folds are the first observable ones, and the cleavage is a penetrative slaty or spaced cleavage we would assume that they were synchronous. The difference in timing of folding and cleavage within one event is not favoured as a general hypothesis. The concept that rocks first folded and later became cleaved (after a rotation in strain axes) is considered to be mechanically unsound.

Folding on two axes should be distinguishable from genuine refolding on two counts. Firstly, polyphase folding should be associated with either two clearly-distinguishable fabrics, a slaty cleavage and a cross-cutting crenulation cleavage, or with a folded slaty cleavage. Single-phase constrictional folding would have a single cleavage. Secondly, the experimental work of Ghosh & Ramberg (1968) demonstrated clear differences in scale and wavelength between constrictional folding and refolding. The former should give rise to varied folds of the same wavelength. However refolding patterns should show smaller wavelengths and amplitudes in the secondary structures than in the first structures. This concept is borne out by most analyses of polyphase deformation.

In conclusion it is suggested that any interpretation of geological structures is affected by current theories. The concept that folding in nature takes place on horizontal planes, in layer parallel compression, to form perfect cylindrical folds is based on the convenience of this model in experimental and mathematical studies. This has given rise to an almost-exclusively two-dimensional view of folding despite the attempt of Flinn (1962) to rectify this. The variety of special explanations for regions of folds with non-axial planar cleavage, including our own, is an example of how interpretation of facts can be influenced by a set of concepts.

Acknowledgements—This paper was written while one of us (S. H. Treagus) was in tenure of a Natural Environmental Research Council Research Fellowship.

REFERENCES

Anthony, M. & Wickham, J. 1978. Finite element simulation of asymmetric folding. *Tectonophysics* **47**, 1–47.

Biot, M. A. 1961. Theory of folding of stratified viscoelastic media and its implications in tectonics and orogenesis. *Bull. geol. Soc. Am.* **72**, 1595–1620.

Borradaile, G. J. 1978. Transected folds: a study illustrated with examples from Canada and Scotland. *Bull. geol. Soc. Am.* **89**, 481–493.

Brenchley, P. J. & Treagus, J. E. 1970. The stratigraphy and structure of the Ordovician rocks between Courtown and Kilmichael Point, Co. Wexford. *Proc. R. Ir. Acad.* **69B**, 83–100.

Dieterich, J. H. & Carter, N. L. 1969. Stress history of folding. *Am. J. Sci.* **267**, 129–154.

Dubey, A. K & Cobbold, P. R. 1977. Noncylindrical flexural slip folds in nature and experiment. *Tectonophysics* **38**, 223–239.

Ekren, E. B. & Frischknecht, F. C. 1967. Geological–geophysical investigations of bedrock in the Island Fells Quadrangle, Aroostock and Penobscot Counties, Maine. *United States Geological Survey.* Prof. Paper 527.

Fleuty, M. J. 1964. The description of folds. *Proc. geol. Ass. Lond.* **75**, 461–492.

Flinn, D. 1962. On folding during three-dimensional progressive deformation. *Q. Jl geol. Soc. Lond.* **118**, 385–433.

Flinn, D. 1965. Deformation in metamorphism. In- *Controls of metamorphism* (edited by Pitcher, W. S. & Flinn, G. W.), *Geological Journal Special Issue No. 1*, Oliver & Boyd, Edinburgh & London. 48–72.

Flinn, D. 1978. Construction and computation of three-dimensional progressive deformations. *J. geol. Soc. Lond.* **135**, 291–305.

Ghosh, S. K. & Ramberg, H. 1968. Buckling experiments on intersecting fold patterns. *Tectonophysics* **5**, 89–105.

Hudleston, P. J. 1973. Fold morphology and some geometrical implications of theories of fold development. *Tectonophysics* **16**, 1–46.

Jaeger, J. C. 1964. *Elasticity, Fracture and Flow.* Methuen & Co., London.

Powell, C. McA. 1974. Timing of slaty cleavage during folding of Precambrian rocks, Northwest Tasmania. *Bull. geol. Soc. Am.* **85**, 1043–1060.

Ramberg, H. 1959. Natural and experimental boudinage and pinch and swell structures. *J. Geol.* **63**, 512–526.

Ramberg, H. 1963. Fluid dynamics of viscous buckling applicable to folding of layered rocks. *Bull. Am. Ass. Petrol. Geol.* **47**, 484–505.

Ramberg, H. 1976. The strain in a sheet intersecting the strain ellipsoid at any angle. *Bull. Soc. géol. France* (7), **18**, 1417–1422.

Ramberg, H. & Ghosh, S. K. 1977. Rotation and strain of linear and planar structures in three-dimensional progressive deformation. *Tectonophysics* **40**, 309–337.

Ramsay, D. M. 1979. Analysis of rotation of folds during progressive deformation. *Bull. geol. Soc. Am.* **90**, 732–738.

Ramsay, D. M. & Sturt, B. A. 1973. An analysis of noncylindrical and incongruous fold patterns from the EoCambrian of Söröy northern Norway. I Non-cylindrical, incongruous and aberrant folding. *Tectonophysics* **18**, 81–107.

Ramsay, J. G. 1965. Structural investigations in the Barberton mountain land, Eastern Transvaal. *Trans. geol. Soc. S. Afr.* **66**, 353–401.

Ramsay, J. G. 1967. *Folding and fracturing of rocks.* McGraw Hill, New York.

Ramsay, J. G. & Wood, D. S. 1973. The geometrical effects of volume change during deformation processes. *Tectonophysics* **16**, 263–277.

Roberts, D. 1971. Abnormal cleavage patterns in fold hinge zones from Varanger peninsula, Northern Norway. *Am. J. Sci.* **271**, 170–180.

Rust, B. 1965. The stratigraphy and structure of the Whithorn area of Wigtownshire, Scotland, *Scott. J. Geol.* **1**, 101–133.

Sanderson, D. J. 1973. The development of fold axes oblique to the regional trend. *Tectonophysics* **16**, 55–70.

Sanderson, D. J., Andrews, J. R., Phillips, W. E. A. & Hutton, D. H. W. 1980. Deformation Studies in the Irish Caledonides. *J. geol. Soc. Lond.* **137**, 289–302.

Shimamoto, T. & Hara, I. 1976. Geometry and strain distribution of single-layer folds. *Tectonophysics* **30**, 1–34.

Soper, N. J. & Moseley, F. 1978. Structure. In: *The Geology of the Lake District* (edited by Moseley, F.). Yorkshire Geol. Soc. occasional publication No. 3, 45–67.

Stringer, P. 1975. Acadian slaty cleavage non coplanar with fold axial surfaces in the Northern Appalachians. *Can. J. Earth Sci.* **12**, 949–961.

Stringer, P. & Treagus, J. E. 1980. Non-axial planar S_1 cleavage in the Hawick Rocks of the Galloway area, Southern Uplands, Scotland. *J. Struct. Geol.* **2**, 317–331.

Stringer, P. & Treagus, J. E. Asymmetric folding in the Hawick Rocks of the Galloway area, Southern Uplands. *Scott. J. Geol.* **17**, in press.

Sutton, J. & Watson, J. 1956. The Boyndie syncline of the Dalradian of the Banffshire Coast. *Q. Jl geol. Soc. Lond.* **112**, 103–130.

Treagus, S. H. 1972. Processes in fold development. Unpublished Ph.D. thesis, University of Manchester, England.

Treagus, S. H. 1973. Buckling stability of a viscous single-layer system, oblique to the principal compression. *Tectonophysics* **19**, 271–289.

Treagus, S. H. 1981. A theory of stress and strain variations in viscous layers and its geological implications. *Tectonophysics* **72**, 75–103.

Weir, A. 1968. Structural history of the Silurian rocks on the coast west of Gatehouse, Kirkcudbrightshire. *Scott. J. Geol.* **4**, 31–52.

APPENDIX

I. *Derivation of strain in any plane of known initial attitude in the strain ellipsoid*

The principal axes of the irrotational strain ellipsoid ($X > Y > Z$) are defined as vertical and horizontal in Fig. 15(a). The layering plane is defined by two lines contained in it: s_0 is the initial horizontal line, or strike; d_0 is the initial dip line. After strain, their positions change and are no longer perpendicular. We have chosen to define the horizontal plane as a principal plane, and hence the line s_n will remain the strike of the plane throughout strain. However, d_n is not the direction of dip of plane $s_n d_n$ after strain.

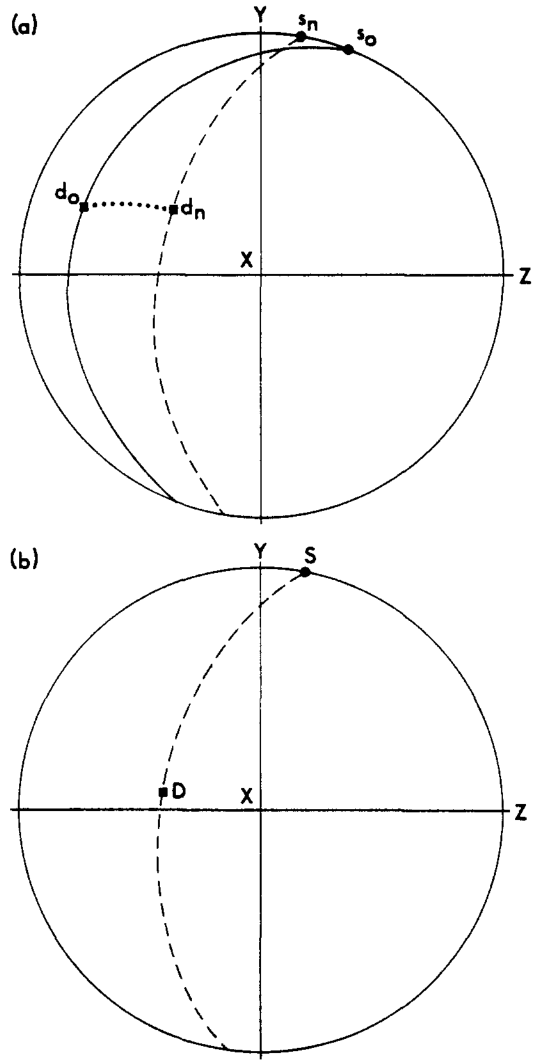

Fig. 15.(a) Orientation convention used in Appendix I. The continuous great circle is the initial layering plane. s_0 and d_0 are the initial strike and dip lines respectively. The broken great circle is the deformed layering plane and s_n and d_n are the deformed strike and dip lines after strain increment n. (b) Orientation convention used in Appendix II. The broken great circle is the deformed layering plane. S and D are the strike and dip lines respectively.

The strains will be written as quadratic elongations λ_1, λ_2 and λ_3 (Ramsay 1967), where $\lambda_1 = X^2$, $\lambda_2 = Y^2$, and $\lambda_3 = Z^2$. The attitude of the layering plane to λ_i is defined by direction cosines. α_i are the direction cosines of s_0 to λ_i, and β_i are the direction cosines of d_0 to λ_i, with $i = 1, 2, 3$. α_i' and β_i' represent the direction cosines after a given strain.

The following equations can be written, following Ramsay (1967), where λ_s and λ_d are the quadratic elongations parallel to s_n and d_n.

$$\lambda_s = \alpha_1^2 \lambda_1 + \alpha_2^2 \lambda_2 + \alpha_3^2 \lambda_3 \tag{1}$$

$$\lambda_d = \beta_1^2 \lambda_1 + \beta_2^2 \lambda_2 + \beta_3^2 \lambda_3 \tag{2}$$

$$\alpha_i'^2 = \frac{\lambda_i}{\lambda_s} \alpha_i^2 \tag{3}$$

$$\beta_i'^2 = \frac{\lambda_i}{\lambda_d} \beta_i^2 . \tag{4}$$

The angle $s_n \wedge d_n$ defined as ϕ' is given by

$$\cos \phi' = \alpha_1' \beta_1' + \alpha_2' \beta_2' - \alpha_3' \beta_3' . \tag{5}$$

This angle can be determined from equations (1) to (4). It is related to the shear strain for s_n and d_n such that

$$\gamma_s = \gamma_d = \frac{1}{\tan \phi} \tag{6}$$

λ_s, λ_d and γ_s may be used to determine the amount and orientation of the principal quadratic elongation ($\lambda_{p1} > \lambda_{p2}$) in the plane $s_n d_n$. The orientation of λ_{p1} is defined by angle θ' to s_n. Quadratic elongations are written below as reciprocals(λ'), following Ramsay (op. cit.).

$$\lambda_s' = \frac{(\lambda_{p2}' + \lambda_{p1}')}{2} - \frac{(\lambda_{p2}' - \lambda_{p1}')}{2} \cos 2\theta' \tag{7}$$

$$\lambda_d' = \frac{(\lambda_{p2}' + \lambda_{p1}')}{2} - \frac{(\lambda_{p2}' - \lambda_{p1}')}{2} \cos 2(\phi' - \theta') \tag{8}$$

$$\gamma_s \cdot \lambda_s' = \frac{(\lambda_{p2}' - \lambda_{p1}')}{2} \sin 2\theta' \tag{9}$$

$$\gamma_d \cdot \lambda_d' = \frac{(\lambda_{p2}' - \lambda_{p1}')}{2} \sin 2(\phi' - \theta') . \tag{10}$$

Thus

$$\frac{\lambda_s}{\lambda_d} = \frac{\sin 2(\phi' - \theta')}{\sin 2\theta'} \tag{11}$$

and by rearrangement

$$\tan 2\theta' = \frac{\sin 2\phi'}{\lambda_s/\lambda_d + \cos 2\phi'} . \tag{12}$$

λ_s, λ_d and ϕ' may be computed from equations (1), (2) and (5), hence θ' determined. Reorganisation of (7) and (8) yields

$$\lambda_{p1} = \frac{\lambda_s}{1 - [(1 - \cos 2\theta')/(\sin 2\theta' \tan 2\phi')]} \tag{13}$$

$$\lambda_{p2} = \frac{\lambda_s}{1 + [(1 - \cos 2\theta')/(\sin 2\theta' \tan 2\phi')]} . \tag{14}$$

These can be computed.

This method was used to determine the changing orientations of suites of planes of known initial attitude in three equal increments of strain. At each stage the values of the two principal elongations in the plane, and their orientations, were computed. The method is applicable to planes of any attitude, and strain ellipsoids of varied shapes. From the relative values of the two principal strains in the plane of layering, it is possible to determine whether folding is active.

II. *Derivation of strain in any plane of known final attitude in the strain ellipsoid*

The notation used for the principal strain ellipsoid and its attitude are defined in I. The layering plane is here defined by its final strike and dip lines S and D (Fig. 15b), which are perpendicular. S is thus parallel to s_n of I, but D is not parallel to d_n.

The attitude of S to λ_i is given by direction cosines ζ_i, and D to λ_i by δ_i, with $i = 1, 2, 3$. The undeformed values are not used in this analysis.

Rewriting equations (1) and (2) for the deformed state,

$$\lambda_S' = \zeta_1^2 \lambda_1' + \zeta_2^2 \lambda_2' + \zeta_3^2 \lambda_3' \tag{15}$$

$$\lambda_D' = \delta_1^2 \lambda_1' + \delta_2^2 \lambda_2' + \delta_3^2 \lambda_3' . \tag{16}$$

The proceeding analysis is simplified by the interdependence of the six direction cosines. The orientation of S is such that $\zeta_1 = 0$, and

$$\delta_3^2 = (1 - \zeta_3^2)(1 - \delta_1^2) . \tag{17}$$

The direction cosines may thus all be written in terms of ζ_3 and δ_1, where ζ_3 is sine the angle of strike and δ_1 is sine the angle of dip. Equation (15) and (16) may thus be written:

$$\lambda_S' = (1 - \zeta_3^2)\lambda_2' + \zeta_3^2 \lambda_3' \tag{18}$$

$$\lambda_D' = \delta_1^2 \lambda_1' + \zeta_3^2 (1 - \delta_1^2) \lambda_2' + (1 - \zeta_3^2)(1 - \delta_1^2) \lambda_3' . \tag{19}$$

Thus, from the value of strike and dip of the deformed layering plane and λ_1', λ_2' and λ_3', the values of λ_S' and λ_D' can be determined. These may be used to calculate the principal reciprocal quadratic elongations ($\lambda_{p1}' > \lambda_{p2}'$) in the plane, where θ' is at angles of λ_{p1}' to S. It can be shown that

$$\lambda_{p1}' = \frac{\lambda_S'(1 + \sec 2\theta') + \lambda_D'(1 - \sec 2\theta')}{2} \tag{20}$$

$$\lambda_{p2}' = \frac{\lambda_S'(1 - \sec 2\theta') + \lambda_D'(1 + \sec 2\theta')}{2} . \tag{21}$$

The angle θ' cannot be determined without another equation. The relationship of shear strain of line S will be used, following Jaeger (1964, p. 36):

$$\gamma_S^2 \cdot \lambda_S'^2 = (\lambda_2' - \lambda_3')^2 \zeta_2^2 \zeta_3^2 + (\lambda_3' - \lambda_1')^2 \zeta_3^2 \zeta_1^2 + (\lambda_1' - \lambda_2')^2 \zeta_1^2 \zeta_2^2 \tag{22}$$

which simplifies, in this analysis, to

$$\gamma_S^2 \cdot \lambda_S'^2 = (\lambda_2' - \lambda_3')^2 \zeta_3^2 (1 - \zeta_3^2) . \tag{23}$$

This is equivalent to the shear of S in the strain ellipse for plane SD, which can be written

$$\gamma_S \lambda_S' = \frac{(\lambda_{p2}' - \lambda_{p1}') \sin 2\theta'}{2} . \tag{24}$$

Equating (23) and (24), and using (20) and (21):

$$\tan 2\theta' = 2 \frac{(\lambda_3' - \lambda_2') \zeta_3 (1 - \zeta_3^2)^{1/2}}{\lambda_D' - \lambda_S'} . \tag{25}$$

Using (18) and (19), θ' can be computed and thus from (20) and (21) λ_{p1} and λ_{p2} determined.

This method is useful to determine strain in planes of known final attitude and unknown initial attitude. It is particularly applicable to single-example problems, and by using strike and dip angles only, the computation is quite simple.